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Scaling Properties of the Giant Dipole Resonance Width in Hot Rotating nuclei

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We study the systematics of the giant dipole resonance width Γ in hot rotating nuclei as a function of temperature T, spin J and mass A. We compare available experimental results with theoretical calculations that include thermal shape fluctuations in nuclei ranging from $A = 45$ to $A = 208$. Using the appropriate scaled variables, we find a simple phenomenological function $\Gamma(A,T,J)$ which approximates the global behavior of the giant dipole resonance width in the liquid drop model. We reanalyze recent experimental and theoretical results for the resonance width in Sn isotopes and ^{208}Pb.

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Hot rotating nuclei are usually produced in heavy ion fusion reactions through transfer of the energy and angular momentum of the relative motion into internal degrees of freedom. The resulting hot nucleus can decay through particle and gamma-ray emission. From the decay patterns of these nuclei one can hope to understand their properties under extreme conditions such as high temperature and spin. A particularly useful experimental probe in the study of hot nuclei has been the giant dipole resonance (GDR) [1,2]. At zero temperature, the GDR vibrational frequency is inversely proportional to the length of the axis along which the vibration occurs, and the quadrupole deformation of the nucleus can be inferred from the splitting of the GDR peak. At finite temperature the nuclear shape fluctuates, and the relationship between the shape and the observed resonance properties is more complex. In the adiabatic limit the observed GDR strength function is calculated through an average over a thermal ensemble of shapes corresponding to all quadrupole degrees of freedom [3]. These include both the intrinsic shape and the nuclear orientation with respect to its rotation axis. The fluctuation theory explains successfully both the observed cross-section and angular anisotropy of the GDR radiation [4–6].

In recent years, a wealth of experimental results for the GDR has become available in wider regions of temperature and spin [6–9]. In the fusion experiments, higher excitation energies are usually accompanied by larger amounts of angular momentum transfer. However, in recent inelastic scattering experiments of light particles (e.g. alpha particles) from heavy nuclei the GDR could be excited over a range of temperatures without substantial angular momentum transfer [9]. Although detailed theoretical analyses of the GDR have been done in many nuclei, a comprehensive study of its global features has been lacking. In this letter we present a systematic analysis of the GDR width as a function of temperature T, spin J and mass A. We compare available experimental results with theoretical calculations in nuclei ranging from $A \sim 45$ to $A \sim 208$. The calculations include thermal shape fluctuations using both Nilsson-Strutinsky (NS) and liquid drop (LD) free energy surfaces. We find that by introducing appropriate scaling of the variables it is possible to approximate the GDR width $\Gamma(A,T,J)$ in the LD regime by a simple phenomenological function.

A theory of hot rotating nuclei was developed in the framework of the Landau theory, where the quadrupole deformation parameters in the laboratory frame $\alpha_{2\mu}$ ($\mu = -2,\ldots,2$) play the role of the order parameters [10]. The free energy at constant temperature T and angular velocity ω is expanded in the form

$$F(T,\omega;\alpha_{2\mu}) = F(T,\omega=0;\beta,\gamma) - \frac{1}{2}\left(\hat\omega\cdot\mathcal{I}\cdot\hat\omega\right)\omega^2 \ , \quad (1)$$

where β,γ are the intrinsic shape parameters. The quantity $\hat\omega\cdot\mathcal{I}\cdot\hat\omega = I_{x'x'}\sin^2\theta\cos^2\phi + I_{y'y'}\sin^2\theta\sin^2\phi + I_{z'z'}\cos^2\theta$ is the moment of inertia about the rotation axis $\hat\omega$, expressed in terms of the principal moments of inertia $I_{x'x'}, I_{y'y'}, I_{z'z'}$ and the Euler angles $\Omega = (\psi,\theta,\phi)$ that describe the nuclear orientation with respect to the rotation axis. In the finite nuclear system, fluctuations in the order parameters are important and the probability of finding the nucleus in a state with deformation $\alpha_{2\mu}$ is given by the Boltzmann factor $\exp[-F(T,\omega;\alpha_{2\mu})/T]$ [11–14]. At high spins it is necessary to project on constant spin [15], and in the saddle point approximation the free energy at spin J is the Legendre transform of (1)

$$F(T,J;\alpha_{2\mu}) = F(T,\omega=0;\beta,\gamma) + \frac{(J+1/2)^2}{2\hat\omega\cdot\mathcal{I}\cdot\hat\omega} \ . \quad (2)$$

In the non-rotating ($\omega = 0$) case, the GDR absorption cross-section at a fixed shape $\alpha_{2\mu}$ is described by a superposition of Lorentzians with centroids inversely proportional to the lengths R_j of the corresponding principal axes: $E_j = E_0(R_0/R_j)$, and widths satisfying a power law $\Gamma_j = \Gamma_0(A)(E_j/E_0)^\delta$ (with $\delta = 1.6$). The R_j depend on the intrinsic shape through the Hill-Wheeler parametrization $R_j = R_0\exp\left[-\sqrt{5/4\pi}\beta\cos(\gamma-2\pi j/3)\right]$. E_0 and $\Gamma_0(A)$ are the mass-dependent energy and width, respectively, for a spherical shape and are assumed to be temperature

independent. For $\omega \neq 0$, the eigenfrequencies $E_j(\omega)$ are affected by the Coriolis force, and it is necessary to transform from the rotating frame to the laboratory frame [13].

At constant temperature and spin, the observed GDR cross-section is calculated by a thermal average of the shape-dependent cross-section [15]

$$\sigma(E_\gamma; J, T) = \frac{1}{Z} \int \mathcal{D}[\alpha] \frac{e^{-F(T,J;\alpha_{2\mu})/T}}{(\hat{\omega} \cdot \mathcal{I} \cdot \hat{\omega})^{3/2}} \sigma(E_\gamma; \omega, \alpha_{2\mu}) \quad (3)$$

where the measure is given by [13] $\mathcal{D}[\alpha] = \beta^4 \mid \sin 3\gamma \mid d\beta d\gamma d\Omega$, and $Z = \int \mathcal{D}[\alpha] \exp[-F/T]/(\hat{\omega} \cdot \mathcal{I} \cdot \hat{\omega})^{3/2}$ is the partition function.

The free energy surface at $\omega = 0$ and the principal moments of inertia are calculated using either an NS approach (which includes shell corrections), or the LD model. At higher temperatures, shell effects melt and both approaches agree well with each other.

We have carried out a comprehensive study of the GDR over a wide range of nuclei for which experimental data exist [7]: ^{45}Sc, ^{59,63}Cu, ^{90}Zr, ^{92,100}Mo, $^{106-120}$Sn, ^{156}Dy, ^{166,168}Er, ^{188}W, and ^{208}Pb. We find that a simple behavior emerges in the LD model. We first examine the spin dependence of the width Γ at fixed temperature T. Fig 1(a) shows Γ versus spin for ^{59,63}Cu. The symbols are the experimental results and the lines are theoretical LD calculations. The overall agreement between theory and experiment is good (except at low T where the LD model does not apply); the width is insensitive to spin for $J \lesssim 20\hbar$ and increases at higher spins. Similarly, in 1(b) we show the spin dependence of width in ^{106}Sn, where our LD calculations at $T = 1.8$ MeV (solid) reproduce well the experimental behavior [8], and account for up to $\sim 20\%$ enhancement at high spins over the calculations of [8] (dashes). For ^{106}Sn the width remains insensitive to spin up to a higher spin of $J \lesssim 30\hbar$. In Fig. 1(c) we show Γ versus J at $T = 2$ MeV for several nuclei in different mass regions for spins up to their respective fission limit. The sensitivity of the GDR width to spin is larger for the lighter nuclei as is expected from their smaller moment of inertia. We have investigated several possible scalings to relate the reduced widths $\Gamma(T, J, A)/\Gamma(T, J = 0, A)$ of various masses. At high spins, the rotational energy $J^2/2I$ dominates. Since for a rigid body $I \propto A^{5/3}$, this suggests a scaling of the spin by $A^{5/6}$. Fig. 1(d-e) shows the reduced width as a function of $\xi \equiv J/A^{5/6}$. At a fixed temperature the reduced width for various masses falls approximately on a single curve. While this is clearly not an exact scaling of the theory described by Eqs. (3) and (2), it provides a rather good approximation. The scaling improves with increasing temperature. We remark that a significant mass dependence of the width is observed when plotted either versus angular velocity ω or the rotation parameter y of the LD [16].

The scaling curves in Fig. 1(e) exhibit a significant temperature dependence. We choose the reduced width $\Gamma(T_0, J, A)/\Gamma(T_0, J = 0, A)$ at $T_0 = 1$MeV to be our 'reference' function $L(\xi)$ with $\xi = J/A^{5/6}$ The reduced widths at different temperatures are related through the power law $[\Gamma(T, J, A)/\Gamma(T, J = 0, A)]^{(T/T_0+3)/4}$ as is shown in Fig. 1(f). Hence the approximate spin dependence of Γ is described by $\Gamma(T, J, A)/\Gamma(T, J = 0, A) \approx [L(\xi)]^{4/(T/T_0+3)}$.

Next we examine the temperature (and mass) dependence of the width at zero spin $\Gamma(T, J = 0, A)$. In the left panel of Fig. 2 we show the experimental width at low spin ($J \lesssim 20\hbar$) for ^{59,63}Cu as a function of T in comparison with exact LD calculations (solid line). The quantity $\Gamma(T, J = 0, A) - \Gamma_0(A)$ (where $\Gamma_0(A)$ is the width for a spherical shape) increases monotonically from zero as a function of T. At high temperatures it behaves as $\sqrt{T}$: using the leading order term $B\beta^2$ for the LD free energy (with B constant), we can remove the temperature dependence in the Boltzmann factor $\exp[-B\beta^2/T]$, by scaling β by $\sqrt{T}$. This works well only for temperatures $T \gtrsim 2$ MeV, and a much better global fit is obtained from $\Gamma(T, J = 0, A) - \Gamma_0(A) \approx c(A)\log(1 + T/T_0)$, where $T_0 = 1$ MeV is the reference temperature and $c(A)$ is a constant depending weakly on A. In the right panel of Fig. 2, we show this fitting function for ^{90}Zr (solid line) and compare it with the liquid drop calculations (squares). The dashed line demonstrates the $\sqrt{T}$ behavior at large T. The function $c(A)$ depends on the the choice of Γ_0, since increasing the width Γ_0 does not result in a constant shift of the width at all temperatures, but rather a modification of the prefactor $c(A)$. A parametrization which seems to work well over the mass range studied (and for our physical choices of Γ_0) is $c(A) \approx 6.45 - A/100$.

We conclude that a good phenomenological formula to describe the global dependence of the LD GDR width on temperature, spin and mass is:

$$\Gamma(T, J, A) = \Gamma(T, J = 0, A) \left[L\left(\frac{J}{A^{5/6}} \right) \right]^{4/[(T/T_0)+3]} \quad (4)$$
$$\Gamma(T, J = 0, A) = \Gamma_0(A) + c(A)\log(1 + T/T_0) \; .$$

$\Gamma_0(A)$ is usually extracted from the measured ground state GDR, and $T_0 = 1$ MeV is a reference temperature. $L(\xi)$ is the scaling function shown in Fig. 1 (f), which can be approximately fitted by $L(\xi) - 1 \approx 1.8\left[1 + e^{(1.3-\xi)/0.2}\right]^{-1}$. Eqs. (4) provide an approximate description of the systematic behavior of the GDR width in nuclei where the LD model is valid, i.e. in nuclei where shell effects are small or at temperatures where shell effects have already melted. In the top panel of Fig. 3 we correlate the theoretical estimates based on (4) with known experimental results [6–9]. In the bottom panel of Fig. 3(b) we show the ratio between the experimental width $\Gamma_{exp}(T, J, A)$ and the 'theoretical' width $\Gamma(T, J = 0, A)$ calculated from (4) as a function of ξ.

The scaling function $L(\xi)$ is seen to be essentially constant for $\xi \lesssim 0.6$ (indicated by dashed line in Fig. 1(f)). Thus the width is approximately spin-independent up to a spin of $J_1 \sim 0.6A^{5/6}$. It is interesting to compare J_1 with the maximal angular momenta $J_{max}(A)$ for which the fission barrier height is still larger than ~ 8 MeV, guaranteeing reasonable stability against fission [16]. We find that for nuclei with $A \gtrsim 200$, $J_{max} \leq J_1$, and there is no significant spin dependence of the GDR width (see e.g. ^{208}Pb in Fig. 1(c)).

Shell corrections can play a role at lower temperatures. Here we focus on two nuclei of recent experimental [9] and theoretical [17] interest, ^{120}Sn and ^{208}Pb. Fig. 4 shows the results of our calculations of the width as a function of temperature using both LD (dotted line) and NS (solid line) free energy surfaces. We have used $\Gamma_0 = 3.8$ MeV [13] for ^{120}Sn and ^{208}Pb. Our results are compared with the recent calculations of Ref. [17], also shown in Fig. 4 (dashes and dot-dashes) [18]. For ^{208}Pb our calculated widths at temperatures above 1 MeV are significantly larger than those of Ref. [17]. Similarly for ^{120}Sn, our calculated widths are larger than those of Ref. [17] at large temperatures even though our assumed Γ_0 (3.8 MeV) is smaller than the one used in Ref. [17] (5 MeV). When compared with our newly calculated widths, the experimental results of Refs. [9] (open diamonds in Fig. 4) show significant deviations. We have re-evaluated the temperatures corresponding to the ^{120}Sn and ^{208}Pb inelastic scattering data, and found new temperatures (solid diamonds in Fig. 4) that are substantially smaller than the values quoted in Refs. [9]. These revised data points are in better agreement with our calculations (except for the two highest temperature points in Sn). For ^{208}Pb they are also in better agreement with the fusion data (shown by x's). With the above revision of both theory and experiment, the new results confirm the conclusions of Ref. [17]: shell effects on the GDR width are negligible in Sn, while the large shell corrections in Pb cause a suppression of the width at low temperatures. Similar suppression of the width due to shell effects can be seen in the calculations for ^{140}Ce in Ref. [13]. In revising the experimental temperatures, we have included the effect of energy lost by particle evaporation prior to γ-decay [1] in both ^{120}Sn and ^{208}Pb by using the computer code Cascade [19] to average over the decay cascades [20]. In addition, in ^{208}Pb we included the effect of the strong shell correction on the temperature, and in ^{120}Sn we assumed the Reisdorf [21] level density, which has a small shell correction and a nearly constant level density parameter [22] $a \approx A/9$ consistent with experiment [23]. All of these corrections reduce the temperatures quoted in Refs. [9].

In conclusion, we have studied the systematics of the GDR width in hot rotating nuclei over a broad range of nuclear masses in the framework of the thermal fluctuation theory. In the liquid drop limit we have found a phenomenological formula that describes well the width behavior as a function of temperature, spin and mass.

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affect substantially the extracted GDR widths.
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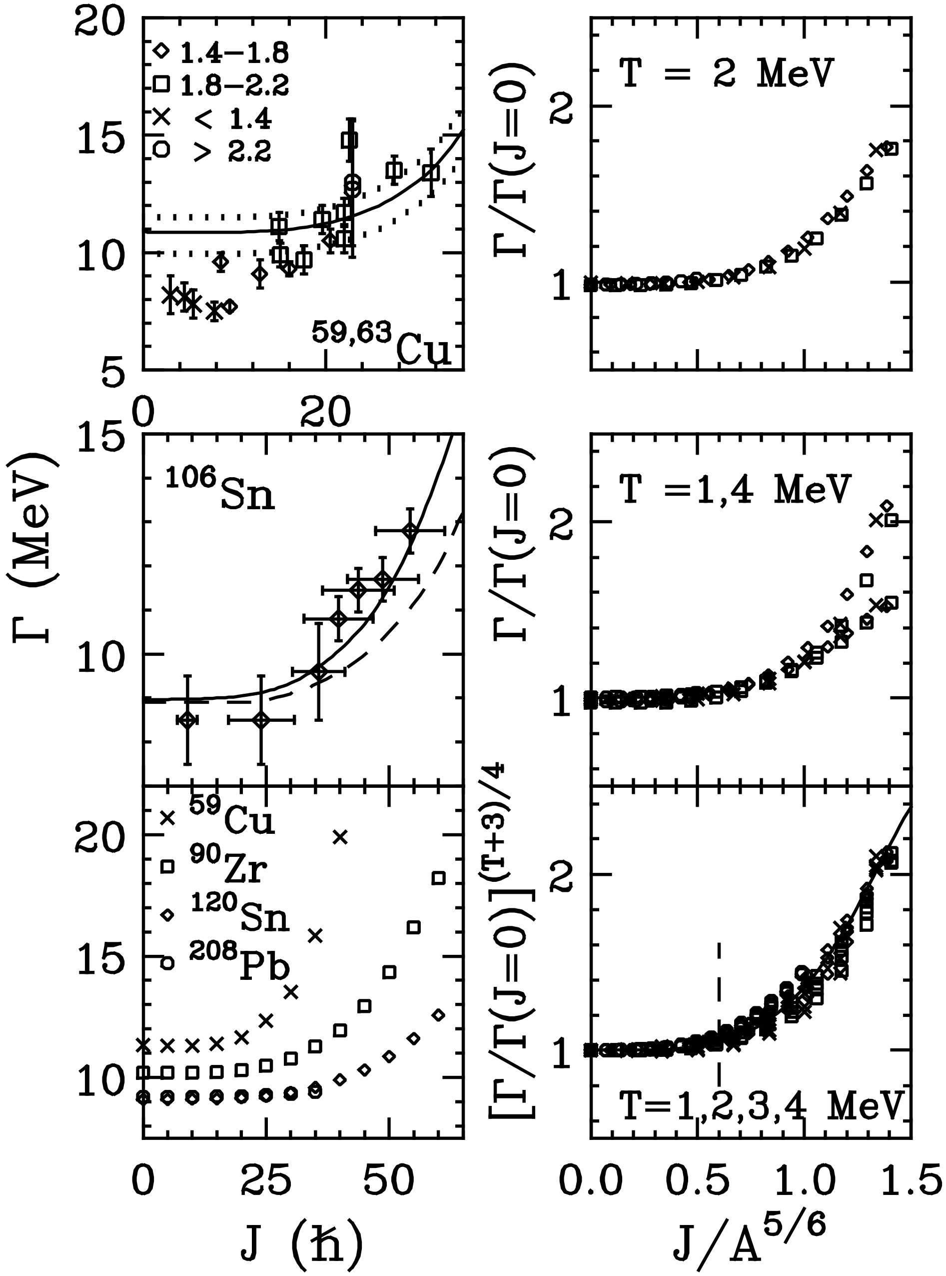
FIG. 1. Spin dependence of the GDR width Γ (left column top to bottom are (a)-(c) and right (d)-(f)). (a) Comparison between experiment (symbols) and LD theory at $T = 1.8$ (solid line) and $T = 1.5, 2.1$ MeV (lower and upper dotted lines, respectively) in ^{59,63}Cu. (b) The experimental widths in ^{106}Sn [8] compared with our LD widths (solid line) and those of [8] (dashes). (c) Systematics of Γ versus spin J (using LD surfaces) at $T = 2$ MeV for ^{59}Cu, ^{90}Zr, ^{120}Sn, and ^{208}Pb. (d) $\Gamma(J, T, A)/\Gamma(T, J = 0, A)$ versus $\xi \equiv J/A^{5/6}$ for all nuclei shown in (c) and for $T = 2$ MeV. (e) Same as (d) but for $T = 1, 4$ MeV displaying temperature dependence at $\xi > 1$. (f) $[\Gamma(T, J, A)/\Gamma(T, J = 0, A)]^{(T/T_0+3)/4}$ versus ξ for $T = 1, 2, 3, 4$ MeV for the nuclei in (c). The solid curve is the scaling function $L(\xi)$ (see text).

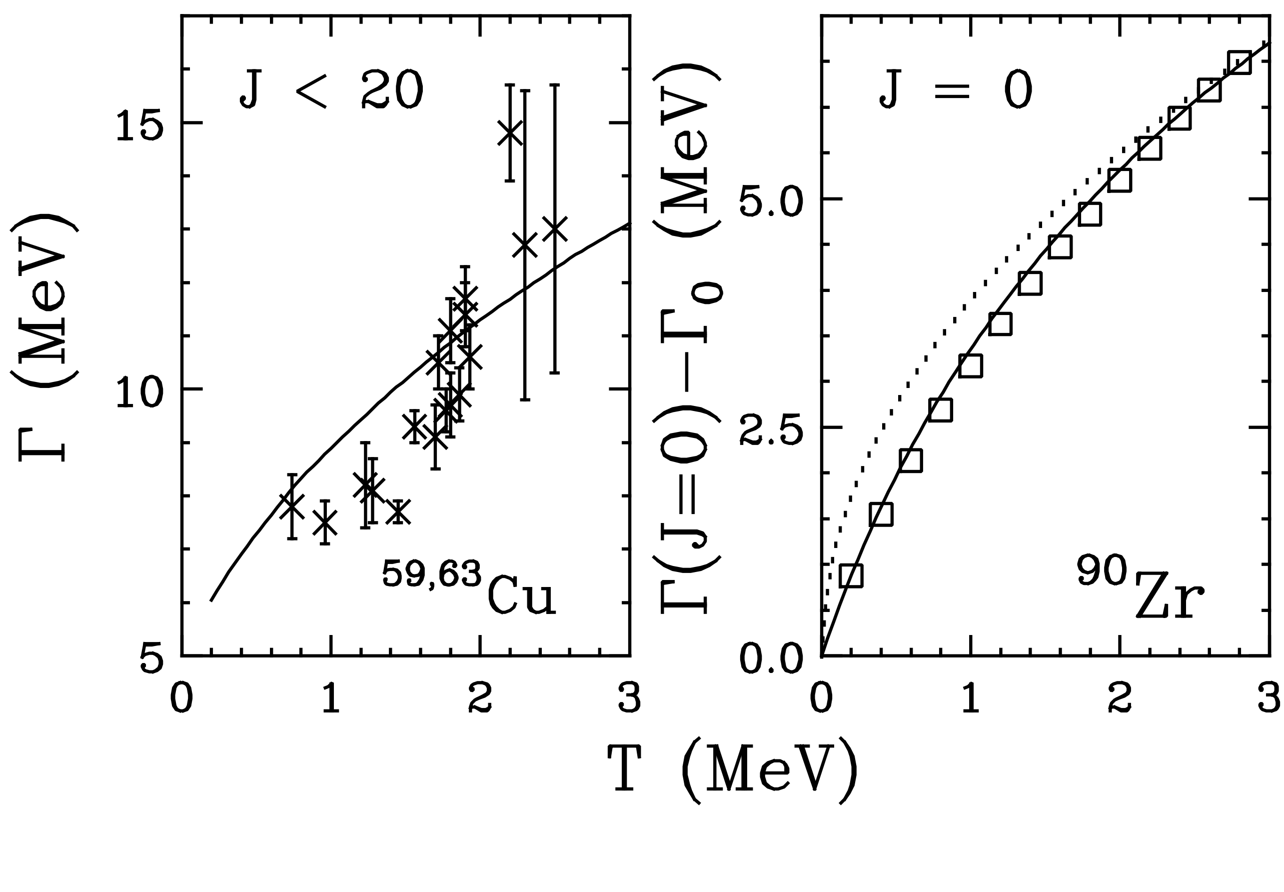
FIG. 2. Temperature dependence of the width. Left: Γ (for $J \lesssim 20\hbar$) as a function of T for ^{59,63}Cu from experiment (symbols) and theory (solid line). Right: $\Gamma(T, J = 0, A) - \Gamma_0$ as a function of T for ^{90}Zr from the LD calculations (boxes), a fit to $c(A)\log(1 + T/T_0)$ (solid line) and a $\sqrt{T}$ behavior (dotted line) which generally fits well at large T.

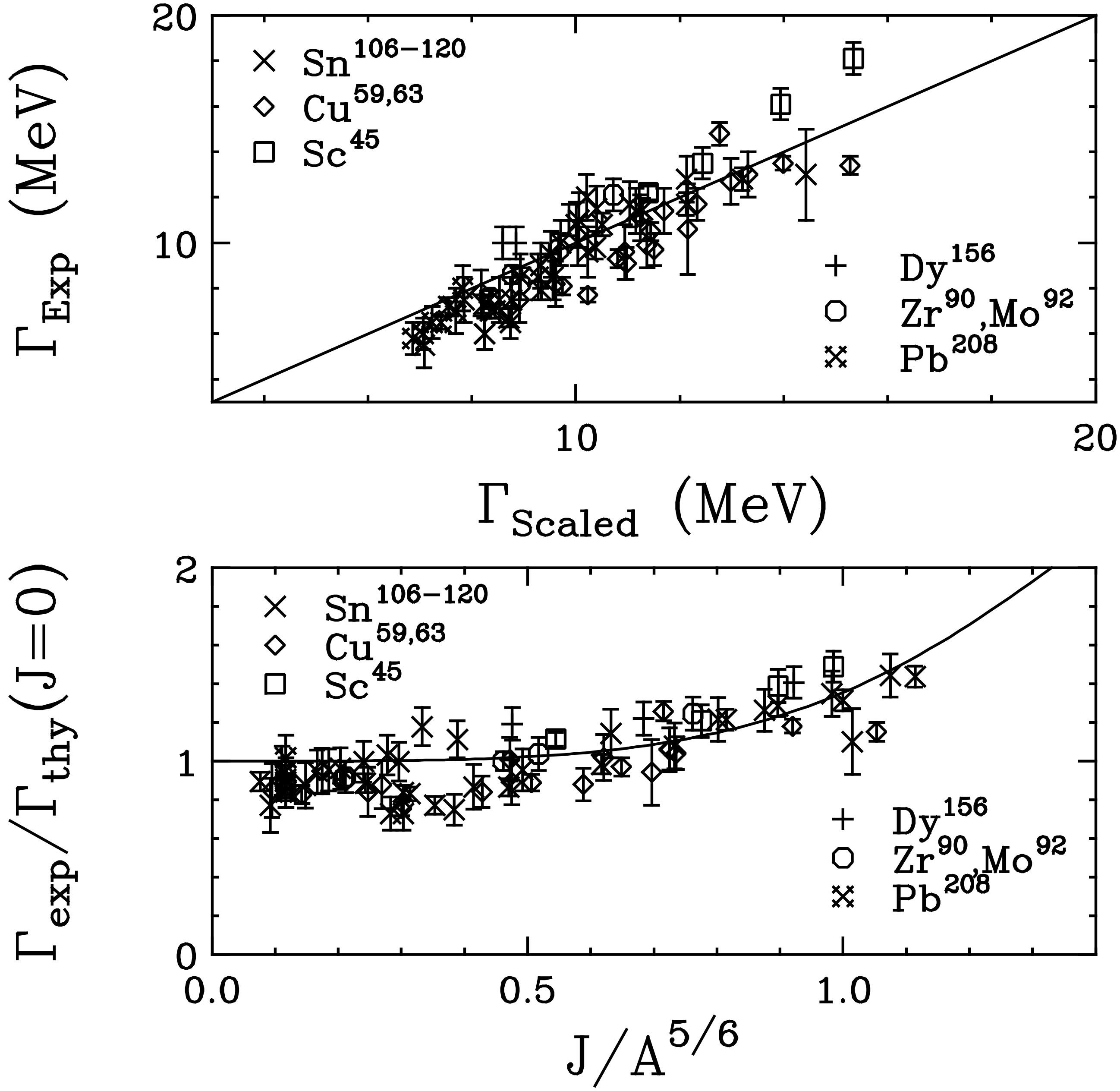
FIG. 3. Comparison of experimental widths with the phenomenological width formula (4). Top: Experimental Γ vs. theoretical scaled Γ for selected nuclei in the mass range $A \sim 45$ to 208. Bottom: Ratio of experimental Γ to theoretical scaled $\Gamma(T, J = 0, A)$ versus $\xi = J/A^{5/6}$. The solid line is the scaling function $L(\xi)$.

FIG. 4. Temperature dependence of the GDR width in ^{120}Sn (left) and ^{208}Pb (right). Top: our calculated widths using NS (solid) and LD (dots) are compared with similar lines calculated in Ref. [17] (dashes are LD and dot-dashes are NS), and with the experimental results of Refs. [9] (open diamonds). Our calculations are the curves giving larger widths at higher temperatures. Bottom: Our theoretical curves compared with the revised data points (solid diamonds). In addition, fusion evaporation data is included (crosses) [7].

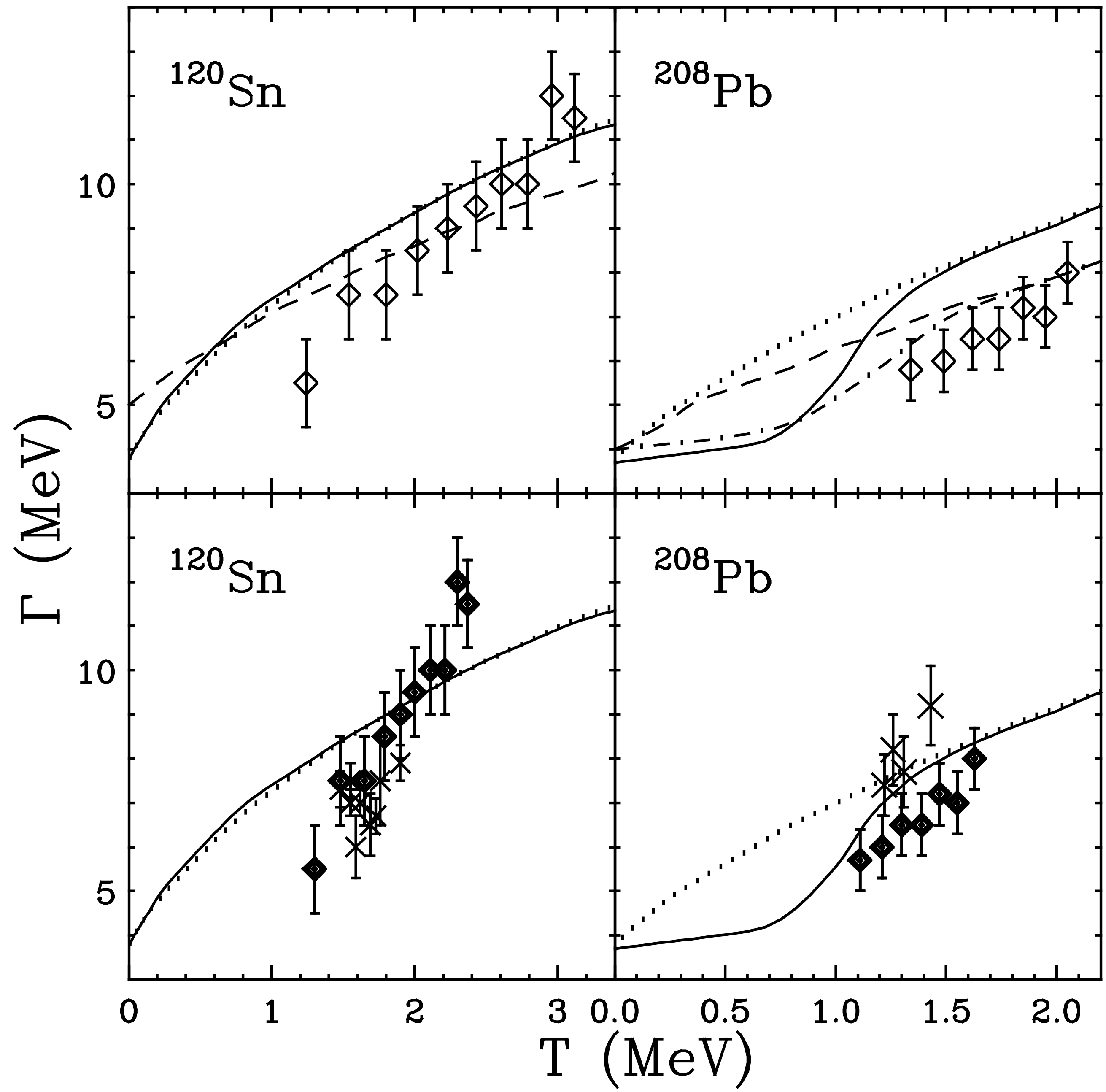


1.4–1.8
1.8–2.2
< 1.4
> 2.2
59,63Cu
106Sn
59Cu
90Zr
120Sn
208Pb
Γ (MeV)
J (ħ)
Γ/Γ(J=0)
[Γ/Γ(J=0)](T+3)/4
T = 2 MeV
T =1,4 MeV
T=1,2,3,4 MeV
J/A5/6


J < 20
^{59,63}Cu
Γ (MeV)
J = 0
^{90}Zr
Γ(J=0)−Γ$_0$ (MeV)
T (MeV)
15
10
5
0
1
2
3
5.0
2.5
0.0



Γ_Exp (MeV)
Γ_Scaled (MeV)
Sn^106−120
Cu^59,63
Sc^45
Dy^156
Zr^90,Mo^92
Pb^208
Γ_exp/Γ_thy(J=0)
J/A^5/6



^{120}Sn
^{208}Pb
^{120}Sn
^{208}Pb
Γ (MeV)
T (MeV)
5
10
0
1
2
3
0.0
0.5
1.0
1.5
2.0